Published by Institute of Physics Publishing for SISSA/ISAS
Received: *July 16, 2003*
Revised: *December 22, 2003*
Accepted: *December 22, 2003*

Comments on noncommutative superspace

Seiji Terashima and Jung-Tay Yee*

Institute for Theoretical Physics, University of Amsterdam
Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands
E-mail: sterashi@science.uva.nl, jungtay@science.uva.nl

Abstract: We study $\mathcal{N} = 1/2$ supersymmetric theory on noncommutative superspace which is a deformation of usual superspace. We consider the deformed Wess-Zumino model as an example and show the vanishing of vacuum energy, the renormalization of superpotential and the non-vanishing of tadpole. We find that the perturbative effective action has terms which are not written in the star deformation. Also we consider gauge theory on the noncommutative superspace and observe that gauge group is restricted. We generalize the star deformation to include noncommutativity between bosonic coordinates and fermionic coordinates.

Keywords: Renormalization Regularization and Renormalons, Superspaces, Non-Commutative Geometry.

*Also at the Korea Institute for Advanced Study, Seoul, Korea.

http://jhep.sissa.it/archive/papers/jhep122003053/jhep122003053.pdf

JHEP12(2003)053

Contents

1. Introduction

Recently, field theories on noncommutative space have been studied extensively. These theories have some similarities to string theory, for example, UV/IR mixing. Indeed, noncommutative field theories can be derived from string theory by the Seiberg-Witten limit [1]. From the dynamics of noncommutative field theories, we expect to understand geometry behind string theory.

As a possible generalization of noncommutative field theories, we can supersymmetrize them. Supersymmetric field theories with noncommutativities only between bosonic coordinates are easy to construct [2, 3]. Then since, in supersymmetric theories, usual bosonic space is extended to superspace which has fermionic coordinates, we are tempted to define noncommutative superspace as a further generalization and to construct field theories on the noncommutative superspace. However, even though there have been many attempts [4]–[15], the construction of the noncommutative superspace with non-anticommuting fermionic coordinates

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} \tag{1.1}$$

with a constant symmetric deformation parameter $C^{\alpha\beta}$ has been known to have serious difficulties. To define analogues of (anti)chiral superfields, we need to define supercharges $Q, \bar{Q}$ and covariant derivatives $D, \bar{D}$. Those operators need to satisfy the Leibnitz rule, which guarantees a product of chiral superfields is again a chiral superfield. However, it is extremely difficult to define such Q, D in the noncommutative superspace because some of them explicitly depend on θ.

JHEP12(2003)053

Very recently, Seiberg showed that we can construct euclidean field theories on noncommutative superspace and these field theories can be derived from the worldvolume theories of D-branes in gravi-photon background [16, 17].[1] It is found out that these field theories have only a half of supersymmetries compared with the field theories without background or deformation.

In this paper, we consider classical and perturbative quantum aspects of field theories defined on noncommutative superspace. We find there are new kinds of (anti-)chiral superfields in the $\mathcal{N}=1/2$ supersymmetric theories and quantum fluctuations generate superpotential of those superfields in the effective action even though the original action does not contain such terms.

The plan of this paper is as follows: In section 2 we discuss new kinds of (anti)chiral superfields in the $\mathcal{N}=\frac{1}{2}$ supersymmetric theory. We show that, although the notion of holomorphicity is violated, the notion of anti-holomorphicity still survives in $\mathcal{N}=\frac{1}{2}$ supersymmetric theories. This anti-holomorphicity leads to the non-renormalization theorem of anti-superpotential and the vanishing of vacuum energy for the deformed Wess-Zumino model. We also consider gauge theories on the noncommutative superspace and show that gauge group is restricted to products of $\mathrm{U}(N)$. We also show that $\mathrm{U}(1)$ sector of $\mathrm{U}(N)$ gauge group is not decoupled from $\mathrm{SU}(N)$ sector. In section 3, we explicitly consider perturbative dynamics of the deformed Wess-Zumino model. We obtain terms like $\int d^2\theta\Phi Q^2\Phi$ with divergent coefficient which was not present in the original deformed Wess-Zumino model. Section 4 is on discussions.

2. $\mathcal{N}=1/2$ supersymmetric theories

According to [16][2], we consider the following deformed superspace

$$\begin{aligned}
\{\hat\theta^\alpha,\hat\theta^\beta\} &= C^{\alpha\beta}\,, &\qquad \{\bar\theta^{\dot\beta},\hat\theta^\alpha\} &= \{\bar\theta^{\dot\beta},\bar\theta^{\dot\beta}\} = [\bar\theta^{\dot\beta},\hat y^\mu]=0\,,\\
[\hat y^\mu,\hat y^\nu] &= i\Theta^{\mu\nu}\,, &\qquad [\hat y^\mu,\hat\theta^\alpha] &= \Psi^{\mu\alpha}\,,
\end{aligned} \tag{2.1}$$

where $\hat\theta$ and $\hat y^\mu\equiv\hat x^\mu+i\hat\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$ are operators. Note that θ is not complex conjugate of $\bar\theta$.

A function of $\hat\theta,\hat y$ should be ordered. In this paper we will always use the Weyl ordered expression. Using a Fourier transformation, it is written as

$$\hat f(\hat y,\hat\theta)=\int d^4k\int d^2\pi e^{-ik\hat y-\pi\hat\theta}\tilde f(k,\pi)\,. \tag{2.2}$$

Then we can have a one to one map between a function of $\hat\theta,\hat y$ to a function of ordinary (anti)commutative coordinates θ,y via

$$f(y,\theta)=\int d^4k\int d^2\pi e^{-iky-\pi\theta}\tilde f(k,\pi)\,. \tag{2.3}$$

[1]It was first found by Ooguri-Vafa [11, 12] that string theories in gravi-photon backgrounds give rise to noncommutative superspaces.

[2]We follow the notations of [16]. In particular, we use lorentzian notations although we only consider the Euclidean theory.

JHEP12(2003)053

A product $\hat{f}_1(\hat{y},\hat{\theta})\,\hat{f}_2(\hat{y},\hat{\theta})$ is easy to compute:

$$\hat{f}_1(\hat{y},\hat{\theta})\,\hat{f}_2(\hat{y},\hat{\theta}) = \int d^4k_1 d^4k_2 \int d^2\pi_1 d^2\pi_2 e^{-i(k_1+k_2)\hat{y}-(\pi_1+\pi_2)\hat{\theta}} e^{i\Delta} \tilde{f}_1(k_1,\pi_1)\tilde{f}_2(k_2,\pi_2)\,, \tag{2.4}$$

where

$$e^{i\Delta} = e^{\frac{1}{2}(-\pi_1 C\pi_2 - ik_1\Theta k_2 - k_1\Psi\pi_2 + k_2\Psi\pi_1)}, \tag{2.5}$$

and

$$\pi_1 C\pi_2 = (\pi_1)_\alpha C^{\alpha\beta}(\pi_2)_\beta\,, \qquad k_1\Theta k_2 = (k_1)_\mu \Theta^{\mu\nu}(k_2)_\nu\,, \qquad k_1\Psi\pi_2 = (k_1)_\mu\Psi^{\mu\alpha}(\pi_2)_\alpha\,. \tag{2.6}$$

Now we define a star product between ordinary functions in the momentum representation as follows :

$$f_1(y,\theta) * f_2(y,\theta) \equiv \int d^4k_1 d^4k_2 \int d^2\pi_1 d^2\pi_2 e^{-i(k_1+k_2)y-(\pi_1+\pi_2)\theta} e^{i\Delta} \tilde{f}_1(k_1,\pi_1)\tilde{f}_2(k_2,\pi_2)\,. \tag{2.7}$$

We can see that $\hat{f}_1(\hat{y},\hat{\theta})\,\hat{f}_2(\hat{y},\hat{\theta})$ is mapped to $f_1(y,\theta)*f_2(y,\theta)$. By the change of integration variables $k = k_1 + k_2, k' = k_1 - k_2$ and $\pi = \pi_1 + \pi_2, \pi' = \pi_1 - \pi_2$, (2.4) becomes a form of (2.2) from which we can identify a corresponding function $f_1 * f_2$ of ordinary coordinates.

We can see the star product is associative just from the definition as in the usual Moyal star product for the bosonic noncommutativity ($C = \Psi = 0$ in our notation). For $\Theta = \Psi = 0$ case, the star product is the same as the fermionic star product defined in [18, 16], which is given by

$$f_1(\theta) * f_2(\theta) = e^{-\frac{1}{2}C^{\alpha\beta}\frac{\partial}{\partial\theta_1^\alpha}\frac{\partial}{\partial\theta_2^\beta}} f_1(\theta_1)f_2(\theta_2)\Big|_{\theta_1=\theta_2=\theta}\,. \tag{2.8}$$

In general, the star product becomes

$$\begin{aligned} f_1(y,\theta) * f_2(y,\theta) &= \\ &= e^{\frac{1}{2}\left(-\frac{\partial}{\partial\theta_1}C\frac{\partial}{\partial\theta_2}+i\frac{\partial}{\partial y_1}\Theta\frac{\partial}{\partial y_2}-i\frac{\partial}{\partial y_1}\Psi\frac{\partial}{\partial\theta_2}+i\frac{\partial}{\partial y_2}\Psi\frac{\partial}{\partial\theta_1}\right)} f_1(y,\theta_1)f_2(y,\theta_2)\Big|_{y_1=y_2=y,\ \theta_1=\theta_2=\theta}\,. \end{aligned} \tag{2.9}$$

Now we can construct field theories on the noncommutative superspace by just replacing ordinary products of superfields to the star products (2.9). But we should be a little bit more careful about supercharges $Q, \bar{Q}$ and covariant derivatives $D, \bar{D}$. In $y, \theta, \bar{\theta}$ coordinates system, we are allowed to define supercharges and covariant derivatives as

$$\begin{aligned} Q_\alpha &= \frac{\partial}{\partial\theta^\alpha}\,, \\ \bar{Q}_{\dot\alpha} &= -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} + 2i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}\,, \\ D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\frac{\partial}{\partial y^\mu}\,, \\ \bar{D}_{\dot\alpha} &= -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}}\,. \end{aligned} \tag{2.10}$$

However, we should note that $\bar{Q}_{\dot{\alpha}}$ is not a linear operator and it does not generate a symmetry. The reason is because θ in the definition can not be considered as a constant any more.

We also can define chiral and anti-chiral superfields in the star product formalism. Chiral superfields are defined as $\bar{D}_{\dot{\alpha}}\Phi(y,\theta,\bar{\theta}) = 0$, which can be rewritten as $\Phi(y,\theta,\bar{\theta}) = \Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F$. Anti-chiral superfields are defined as $D_\alpha\bar{\Phi}(y,\theta,\bar{\theta}) = 0$, which again are written in terms of component fields as $\bar{\Phi}(y,\theta,\bar{\theta}) = \bar{\Phi}(\bar{y},\bar{\theta}) = \bar{A}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) + \bar{\theta}\bar{\theta}\bar{F}$. Here $\bar{y} \equiv y^\mu - 2i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$. There is no ambiguity in the above expressions of $\Phi(y,\theta)$ and $\bar{\Phi}(\bar{y},\bar{\theta})$. From these superfields, we can always construct a lagrangian as

$$L = \int d^4\theta K(\Phi,\bar{\Phi})_* + \int d^2\theta W(\Phi)_*|_{\bar{\theta}=0} + \int d^2\bar{\theta}\bar{W}(\bar{\Phi})_*|_{\theta=0}\,. \tag{2.11}$$

All products are star products. The action is given by $\int d^4x L(x)$. Here we note that $\int d^4x Q(x,\theta,\bar{\theta}) = \int d^4x Q(y,\theta,\bar{\theta})$ since the difference is the integration of a total divergence.

2.1 Chiral superfields in $\mathcal{N}=1/2$ superspace

In this subsection, we first forget about the star product and consider $\mathcal{N}=1/2$ supersymmetric theory on usual (anti)commutative superspace. From a chiral superfield Φ, we can construct other chiral superfields by multiplication, $\Phi * \Phi$ or by differentiation $\partial_\mu\Phi$. Interestingly, if we keep only the $\mathcal{N}=1/2$ supersymmetry, we can construct new kinds of chiral superfield from Φ. $Q_\alpha\Phi$ and $Q^2\Phi$ are chiral super fields since $\bar{D}_{\dot{\beta}}(Q_\alpha\Phi) = 0$ and $Q_\beta(Q_\alpha\Phi) = -Q_\alpha(Q_\beta\Phi)$. From the anti-chiral superfield $\bar{\Phi}$, we can construct anti-chiral superfields $Q\bar{\Phi}, \bar{\theta}_{\dot{\beta}}\bar{\Phi}$ and $\bar{\theta}\bar{\theta}\bar{\Phi}$. They vanish by acting D on them and $\{Q,\bar{\theta}\} = \{Q,Q\} = 0$. Other chiral or anti-chiral superfields can be rewritten in terms of these basic superfields. For example, $Q^2\bar{\Phi}$ can be rewritten using $\bar{\theta}\bar{\theta}\bar{\Phi}$ as $Q^2\bar{\Phi} \sim \Box(\bar{\theta}\bar{\theta}\bar{\Phi})$. There are no other new kinds of chiral nor anti-chiral superfields.

Now we briefly mention several novel features of the $\mathcal{N}=1/2$ superspace. An interesting property of $\mathcal{N}=1/2$ superfields is that F and $\bar{A}$ are invariant themselves under the $\mathcal{N}=1/2$ supersymmetry transformation, which is generated by Q. This property is related to the fact that $F = Q^2\Phi$ and $\bar{\theta}\bar{\theta}\bar{A} = \bar{\theta}\bar{\theta}\bar{\Phi}$ are superfields. Another interesting feature of the $\mathcal{N}=1/2$ superfields is that there are supersymmetric invariants without the integration of spacetime. Polynomials constructed from arbitrary multiplications of $\left(\int d^2\theta G(\Phi_i)\right)|_{\bar{\theta}=0}$ and $\left(\int d^2\bar{\theta}(\bar{\theta}\bar{\theta})\bar{G}(\bar{\Phi}_j)\right)$ are supersymmetric invariants since the supertransformation of these does not have derivative terms. Then using these polynomials, we can construct $\mathcal{N}=1/2$ supersymmetric action which contains terms with arbitrary number of $\int d^2\theta$ and $\int d^2\bar{\theta}$. Or if this type of construction looks unsatisfactory, since there are almost trivial identities

$$\left(\int d^2\theta G(\Phi_i(y,\theta))\right)|_{\bar{\theta}=0}\left(\int d^2\theta' G'(\Phi_i(y,\theta'))\right)|_{\bar{\theta}=0} =$$
$$= \left(\int d^2\theta (Q^2 G(\Phi_i(y,\theta)))G'(\Phi_i(y,\theta))\right)|_{\bar{\theta}=0}\,,$$

$$\left(\int d^2\bar\theta(\bar\theta\bar\theta)\bar G(\bar\Phi_i(\bar y,\bar\theta))\right)\left(\int d^2\bar\theta'(\bar\theta'\bar\theta')\bar G'(\bar\Phi_i(\bar y,\bar\theta'))\right)=$$
$$=\int d^2\bar\theta(\bar\theta\bar\theta)\bar G(\bar\Phi_i(\bar y,\bar\theta))\bar G'(\bar\Phi_i(\bar y,\bar\theta))\,,$$
$$\left(\int d^2\bar\theta(\bar\theta\bar\theta)\bar G(\bar\Phi_i(\bar y,\bar\theta))\right)\left(\int d^2\theta' G'(\Phi_i(y,\theta'))\right)|_{\bar\theta=0}=$$
$$=\int d^2\theta d^2\bar\theta(\bar\theta\bar\theta)\bar G(\bar\Phi_i(\bar y,\bar\theta))G'(\Phi_i(y,\theta))\,, \tag{2.12}$$

we are allowed to change a product of these unusual supersymmetric invariants to a single integration of θ and $\bar\theta$. Anyway, note that this kind of construction is not possible for usual $\mathcal{N}=1$ supersymmetric theories, but unique to $\mathcal{N}=1/2$ theories.

In summary, we can construct $\mathcal{N}=1/2$ supersymmetric lagrangian on commutative superspace as follows. First, we consider a lagrangian on usual $\mathcal{N}=1$ commutative superspace and next add terms (for example, $\mu\int d^2\bar\theta\ \bar\theta\bar\theta\bar\Phi^2=\mu\bar A^2$) constructed with new kinds of chiral and anti-chiral superfields to the $\mathcal{N}=1$ lagrangian. Since additional terms break $\mathcal{N}=1$ supersymmetry to $\mathcal{N}=1/2$ supersymmetry, we have $\mathcal{N}=1/2$ supersymmetric theory on usual (anti)commutative superspace. Generically, this $\mathcal{N}=1/2$ supersymmetric theory has no relation to the deformation (2.1) nor the noncommutative superspace.

Now let us consider the lagrangian on the noncommutative superspace (2.11). It is important to notice that the star products (2.8) and (2.9) can be written in terms of $Q_\alpha(=\frac{\partial}{\partial\theta^\alpha})$ and ∂_μ. This means that, using those new kinds of superfields, we can rewrite (2.11) to the lagrangian on usual (anti)commutative superspace without the star product. For example, the $C^{\alpha\beta}$ dependent term of the Wess-Zumino lagrangian in [16] is given by

$$\frac{1}{3}g\,\mathrm{det}CF^3=\frac{1}{48}g\,\mathrm{det}C\int d^2\theta\Phi(Q^2\Phi)(Q^2\Phi)\,. \tag{2.13}$$

Thus the set of the theories on noncommutative superspace is considered as a special subset of $\mathcal{N}=1/2$ supersymmetric theory.

We will see later that quantum fluctuations of the deformed Wess-Zumino model of [16] generate the term $\int d^2\theta\Phi Q^2\Phi$ to the effective lagrangian. This term can not be obtained from the star deformation of any $\mathcal{N}=1$ supersymmetric theory. The question about the characterization of noncommutative superspace in terms of $\mathcal{N}=1/2$ superspace will be left as a future problem.

Another interesting question we can address is the non-renormalization theorem for $\mathcal{N}=1/2$ supersymmetric theories. If we report the result in advance, non-renormalization theorem and anti-holomorphicity survives for anti-superpotential, but holomorphicity is violated for superpotential and superpotentials are renormalized. To prove the non-renormalization theorem for $\mathcal{N}=1$ supersymmetric theories, we can use the argument by Seiberg and Intriligator[19] : promote coupling constants to chiral or anti-chiral superfields and use the notion of holomorphicity. In order to apply this argument to $\mathcal{N}=1/2$ supersymmetric theories, it is important to note that the superpotential W is not distinguishable from the Kähler potential K. This is because $\bar\theta\bar\theta\sim\delta(\bar\theta)$ is an anti-chiral superfield and a Kähler term $\int d^4x\int d^4\theta\ (\bar\theta\bar\theta)\tilde K(\Phi(y,\theta),\bar\Phi(\bar y,\bar\theta))$ can be converted to a superpotential

$\int d^4x \int d^2\theta \tilde{K}(\Phi(x,\theta), \bar{A}(x))$. This feature is related to the fact that $\bar{A}(x)$ is invariant under the supersymmetry transformation. The lowest component of the anti-chiral superfield $\bar{A}$ itself can appear in the superpotential. Consequently we expect the non-renormalization theorem for the superpotential is no longer valid. Indeed we will see explicitly in the next section that there are quantum corrections to the superpotential of the deformed Wess-Zumino model. On the other hand, the anti-superpotential is distinguished from the Kähler potential and the superpotential. Thus we can safely use the notion of anti-holomorphicity for anti-superpotential.

Another interesting quantity to consider in supersymmetric theories is vacuum energy. Since we have no $\bar{Q}$ in the $\mathcal{N}=1/2$ supersymmetric theory, we can not conclude vacuum energy is zero from the algebraic relation $P = \{Q, \bar{Q}\}$. However, we can argue the deformed Wess-Zumino model has vanishing vacuum energy by the still-remaining $\mathcal{N}=1/2$ supersymmetry. This is because vacuum energy is represented by an anti-superpotential

$$\int d^4y \int d^2\bar{\theta}(\bar{\theta}\bar{\theta})\Lambda_0 = \int d^4y \Lambda_0 \,. \tag{2.14}$$

The deformation parameter $C^{\alpha\beta}$ enters into the lagrangian as $g\det C$. Hence g and $C^{\alpha\beta}$ can be considered as the lowest components of chiral superfields which can not appear in the anti-superpotential. Considering the the fact that the vacuum energy is zero for $C^{\alpha\beta}=0$ case, we can argue that the vacuum energy of the deformed Wess-Zumino model must be zero in all order in perturbation theory. Furthermore, we can conclude that anti-superpotential of the deformed Wess-Zumino model is not renormalized since $C^{\alpha\beta}$ can be considered as the lowest component of a chiral superfield.

2.2 Gauge theory on noncommutative superspace and restriction of the gauge group

Now we consider vector superfields in the noncommutative superspace following [16]. The gauge symmetry acts on the vector superfields as

$$\left(e^V\right)_* \to \left(e^{V'}\right)_* = \left(e^{-i\bar{\Lambda}}\right)_* * \left(e^V\right)_* * \left(e^{i\Lambda}\right)_* \,, \tag{2.15}$$

where $*$ in the $(e^V)_*$ means that all the product in the exponential are understood as the star products. The chiral and anti-chiral field strength superfields are given by

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar{D}\bar{D}\left(e^{-V}\right)_* * D_\alpha \left(e^V\right)_* \\ \bar{W}_{\dot{\alpha}} &= \frac{1}{4}DD\left(e^V\right)_* * \bar{D}_{\dot{\alpha}}\left(e^{-V}\right)_* \,. \end{aligned} \tag{2.16}$$

The gauge transformation for them are

$$\begin{aligned} W_\alpha &\to \left(e^{-i\Lambda}\right)_* * W_\alpha * \left(e^{i\Lambda}\right)_* \,, \\ \bar{W}_{\dot{\alpha}} &\to \left(e^{-i\bar{\Lambda}}\right)_* * \bar{W}_{\dot{\alpha}} * \left(e^{i\bar{\Lambda}}\right)_* \,. \end{aligned} \tag{2.17}$$

From these superfields, we can construct the lagrangian on noncommutative superspace [16].

JHEP12(2003)053

We regard V as a matrix valued vector superfield in order to consider non-abelian gauge theory. In this case, the gauge group is restricted by the requirement of the consistency of the gauge transformation as is the case in the bosonic noncommutative gauge theory [3]. Let us be more specific. We denote $W_\alpha = T^a W_\alpha^a$, where T^a is a $d_r \times d_r$ matrix for a representation R of some gauge group G and satisfies $(T^a)^\dagger = T^a$ and $\mathrm{Tr}(T^a T^b) = k$. The infinitesimal version of the gauge transformation is

$$\delta W_\alpha = -i[\Lambda, W_\alpha] = -\frac{i}{2}[T^a, T^b](\Lambda^a * W_\alpha^b + W_\alpha^b * \Lambda^a) - \frac{i}{2}\{T^a, T^b\}(\Lambda^a * W_\alpha^b - W_\alpha^b * \Lambda^a) \,. \tag{2.18}$$

Note that the last term does not vanish because of non-(anti)commutativity. Therefore the gauge transformation does not close and it is inconsistent if $\{T^a, T^b\}$ is not a linear combination of T^d for some a, b. This means T^a spans a basis of the complex algebra of the $d_r \times d_r$ complex matrices. Thus it is obvious that the gauge transformation is consistent only for the unitary group $G = \mathrm{U}(N)$ or its direct product $G = \prod_a \mathrm{U}(N_a)^{(a)}$. Here we should take T^a as the matrix for the fundamental or anti-fundamental representation. Indeed the fundamental or anti-fundamental representation of the $\mathrm{U}(N)$ spans the whole space of $N \times N$ complex matrices.

Moreover, the representations of the gauge group G of the matter is also restricted to fundamental ($\mathbf{N}$), anti-fundamental ($\bar{\mathbf{N}}$), adjoint ($\mathbf{N} \times \bar{\mathbf{N}}$) or bi-fundamental ($\mathbf{N} \times \bar{\mathbf{M}}$) from the consideration of the possible form of the Kähler potential of the matter. This is because even though we can use T^a of the fundamental representation for the matter, just as in the bosonic noncommutativity [3], we can not construct a gauge invariant Kähler potential for the other tensor representations, for example ($\mathbf{N} \times \mathbf{N}$). The gauge transformations for the fundamental, anti-fundamental, adjoint and bi-fundamental chiral superfields are given by

$$\begin{aligned}
\Phi &\to (e^{-i\Lambda})_* * \Phi \,, \\
\tilde{\Phi} &\to \tilde{\Phi} * (e^{i\Lambda})_* \,, \\
\Phi_{\mathrm{adj}} &\to (e^{-i\Lambda})_* * \Phi_{\mathrm{adj}} * (e^{i\Lambda})_* \,, \\
\Phi_{N\bar{M}} &\to (e^{-i\Lambda^{(1)}})_* * \Phi_{N\bar{M}} * (e^{i\Lambda^{(2)}})_* \,,
\end{aligned} \tag{2.19}$$

respectively and the lagrangians are given by

$$\begin{aligned}
L_\Phi &= \int d^4\theta \left(\bar{\Phi} * (e^{-V})_* * \Phi\right) \\
L_{\tilde{\Phi}} &= \int d^4\theta \left(\tilde{\Phi} * (e^{V})_* * \bar{\tilde{\Phi}}\right) \\
L_{\Phi_{\mathrm{adj}}} &= \int d^4\theta \frac{1}{k} \mathrm{Tr}\left((e^{V})_* * \bar{\Phi}_{\mathrm{adj}} * (e^{-V})_* * \Phi_{\mathrm{adj}}\right) \\
L_{\Phi_{N\bar{M}}} &= \int d^4\theta \mathrm{Tr}\left((e^{V^{(2)}})_* * \bar{\Phi}_{N\bar{M}} * (e^{-V^{(1)}})_* * \Phi_{N\bar{M}}\right) .
\end{aligned} \tag{2.20}$$

For $\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} = C^{\alpha\beta}$, $[\hat{y}^\mu, \hat{y}^\nu] = [\hat{y}^\mu, \hat{\theta}^\alpha] = 0$ case, we can also see this gauge group restriction from the $C^{\alpha\beta}$-dependent terms in the gauge fixed lagrangian of the component fields [16]

$$-iC^{\mu\nu}\, \mathrm{Tr} F_{\mu\nu} \bar{\lambda}\bar{\lambda} + \det C\; \mathrm{Tr}(\bar{\lambda}\bar{\lambda})^2 \,. \tag{2.21}$$

Here $F_{\mu\nu}$ and $\bar{\lambda}$ are transformed as adjoint representations of the gauge group. However, since the couping of (2.21) is not written by commutators, to cancel the contribution from the gauge transformation of (2.21), the gauge transformation of λ should have a term proportional to $\bar{\lambda}\bar{\lambda}$ [16]. It is inconsistent for any gauge group except $G = \prod_a \mathrm{U}(N_a)^{(a)}$.

Related to this gauge group restriction, we can see that the U(1) sector of the U(N) group is not decoupled from the SU(N) sector since the second term of (2.21) contains $\mathrm{Tr}((\bar{\lambda}_{\mathrm{U}(1)}\bar{\lambda}_{\mathrm{U}(1)})(\bar{\lambda}_{\mathrm{SU}(N)}\bar{\lambda}_{\mathrm{SU}(N)})) = \bar{\lambda}_{\mathrm{U}(1)}\bar{\lambda}_{\mathrm{U}(1)}\mathrm{Tr}(\bar{\lambda}_{\mathrm{SU}(N)}\bar{\lambda}_{\mathrm{SU}(N)})$ which is not zero. Here $\bar{\lambda}_{\mathrm{U}(1)}$ and $\bar{\lambda}_{\mathrm{SU}(N)}$ are the U(1) part and the SU(N) part of the $\bar{\lambda}$ respectively. Furthermore, we find out the U(1) gauge theory without matter is not trivial. This is because the coupling $\mathrm{Tr}F_{\mu\nu}\bar{\lambda}\bar{\lambda} = F_{\mu\nu}\bar{\lambda}\bar{\lambda}$ does not vanish even for U(1) group, even though all fields are adjoint representations of the U(1), i.e. chargeless.

As we did on the deformed Wess-Zumino model, we are tempted to reformulate gauge theories on noncommutative superspace as theories on (anti)commutative superspace, since the star deformations contains Q and ∂ only even if we include the vector multiplets into the lagrangian. However, the gauge symmetry on the noncommutative superspace is different from the gauge symmetry on the (anti)commutative superspace. In the component fields formulation, the two gauge transformations have the same form only after the redefinition of the components fields of the noncommutative vector superfield [16]. This means two gauge symmetry are indeed different. Therefore, the gauge symmetry is not manifest if we simply rewrite gauge theories on noncommutative superspace as gauge theories on (anti)commutative superspace.

Now we briefly comment about Lorentz symmetry, which is the symmetry of rotation of x^i $(i = 1, \cdots, 4)$. It is $\mathrm{SO}(4) = \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ in usual commutative Euclidean space. But, in the noncommutative superspace, one $\mathrm{SU}(2)_L$ sector of the Lorentz symmetry corresponding to undotted $\mathrm{SU}(2)_L$ indices is broken because of the presence of $C^{\alpha\beta}$. Note that another $\mathrm{SU}(2)_R$ sector remains unbroken. We can say that since the supercharges commute with both the translation operators P_μ and the unbroken $\mathrm{SU}(2)_R$ generator of the Lorentz symmetry, the $\mathcal{N} = 1/2$ supersymmetry is decoupled from the space-time symmetry for a generic value of $C^{\alpha\beta}$. However, if we set $C^{\alpha\beta}$ to some special value, U(1) subgroup of the $\mathrm{SU}(2)_L$ remains unbroken and the supersymmetry and the space-time symmetry are not decoupled. For example, if we set $C^{\alpha\beta} = \delta_{\alpha\beta}$, U(1) subgroup $O(a) = \exp(ia\sigma_2) \in \mathrm{SU}(2)_L$ where a is a real parameter is unbroken. Since $O(a)$ is real as we see from $O(a)^T = O(a)^{-1}$, $C^{\alpha\beta}(= \delta_{\alpha\beta})$ is invariant under the transformation generated by $O(a)$ as $O(a)\delta_{\alpha\beta}O(a)^T = \delta_{\alpha\beta}$. Actually, this is nothing but a rotation between θ^1 and θ^2.

Finally we consider the matrix model which is formally equivalent to a field theory on the noncommutative superspace with $C^{\alpha\beta}$ and a non-degenerate noncommutative parameter $\Theta^{\mu\nu}$. The action of the matrix model is given from any field theory on the noncommutative superspace by the following way. First, using one to one map of (2.2) and (2.3), we replace any superfield $G(y,\theta,\bar{\theta})$ in the action to a matrix which depends on anti-commuting parameter $\bar{\theta}$, $\hat{G}(\hat{y},\hat{\theta},\bar{\theta}) = \hat{G}_1(\hat{y},\hat{\theta}) + \hat{G}_2(\hat{y},\hat{\theta})\bar{\theta} + \hat{G}_3(\hat{y},\hat{\theta})\bar{\theta}\bar{\theta}$. Here we regard the operator $\hat{y}$ as an infinite dimensional matrix as we do in the usual bosonic noncommutative field theory and $\hat{\theta}^\alpha$ as $\gamma^\alpha(= \sigma_\alpha)$ which are two dimensional gamma matrices. Now the $\hat{G}_i(\hat{y},\hat{\theta})$

is a $(2\infty) \times (2\infty)$ matrix. The integration $\int d^4y$ is replaced by a trace for $\hat{y}$, $\mathrm{Tr}_{\hat{y}}$. As shown in [18], we can replace $\int d^2\theta$ and $(\cdots)|_{\theta=0}$ to the supertrace of the gamma matrices, $\frac{i}{4}\mathrm{Tr}_\gamma(\sigma_3\cdots)$, and the trace of the gamma matrices, $\frac{1}{2}\mathrm{Tr}_\gamma$, respectively. Then finally we obtain a (super)matrix model. Note that we should consider γ_α as a fermion, i.e. anti-commutes with $\bar{\theta}$ or fermionic component fields.

3. Quantum aspects of the Wess-Zumino model in noncommutative superspace

In this section to see the quantum properties discussed in the previous section clearly, we study the Wess-Zumino model in the noncommutative superspace

$$L = \int d^4\theta \Phi\bar{\Phi} + \int d^2\theta \left(\frac{m}{2}\Phi * \Phi + \frac{g}{3}\Phi * \Phi * \Phi\right) + \int d^2\bar{\theta} \left(\frac{\bar{m}}{2}\bar{\Phi} * \bar{\Phi} + \frac{\bar{g}}{3}\bar{\Phi} * \bar{\Phi} * \bar{\Phi}\right). \quad (3.1)$$

We set $\Theta = \Psi = 0$ to separate the effect of fermionic noncommutativities only. In component fields formulation, the effect of the star deformation is to add F^3 term [16], which means only the scalar potential is affected by the deformation. The potential expressed in components fields is

$$V = -F\bar{F} - mAF - gA^2F - \frac{g}{3}\mathrm{det}CF^3 - \bar{m}\bar{A}\bar{F} - \bar{g}\bar{A}^2\bar{F}\,. \quad (3.2)$$

To eliminate the auxiliary fields F and $\bar{F}$, we need equations of motion

$$\begin{aligned} \bar{F} + mA + g\mathrm{det}CF^2 + gA^2 &= 0\,, \\ F + \bar{m}\bar{A} + \bar{g}\bar{A}^2 &= 0\,. \end{aligned} \quad (3.3)$$

Then the potential (3.2) is expressible as

$$V = V(C_{\alpha\beta} = 0) + \mathrm{det}C\left(\frac{1}{3}g\bar{m}^3\bar{A}^3 + g\bar{g}\bar{m}^2\bar{A}^4 + g\bar{g}\bar{m}\bar{A}^5 + \frac{1}{3}g\bar{g}^3\bar{A}^6\right). \quad (3.4)$$

In this paper, since $\bar{g}$ does not appear in the N-point functions of Φs at 1-loop level, we take $\bar{g} \to 0$ limit to understand the essential physics of noncommutative superspace avoiding unnecessary complexities. Then since $\bar{\Phi}$ is free, we can integrate out $\bar{\Phi}$ from (3.1), leaving only terms containing Φ [22].

$$S = \int d^4yd^2\theta \left(\frac{1}{2}\Phi(y,\theta)\left(m - \frac{\Box}{\bar{m}}\right)\Phi(y,\theta) + \frac{1}{3}g\Phi(y,\theta) * \Phi(y,\theta) * \Phi(y,\theta)\right). \quad (3.5)$$

Note that $\int \Phi * \Phi = \int \Phi\Phi$. Since the representation of $*$-operation in momentum superspace is much simpler than its position space representation, it is convenient to use momentum space Feynman rules to calculate quantum corrections. Superfield in momentum space is defined as

$$\Phi(p,\pi) = \int d^4yd^2\theta \exp(ipy + \pi\theta)\Phi(y,\theta)\,. \quad (3.6)$$

Expanded into components fields, the momentum space superfield is expressible as

$$\Phi(p,\pi) = \frac{1}{4}\pi\pi A(p) + \frac{1}{\sqrt{2}}\pi\psi(p) + F(p)\,. \quad (3.7)$$

JHEP12(2003)053

The quantization of the action (3.5) is straightforward. One nice property of the action (3.5) is that separate treatment between bosonic coordinates and fermionic coordinates is possible. Since we have only $\Phi(y, \theta)$ in the action, we don't need chiral projectors any more. For bosonic coordinates, Feynman rules are nearly the same as the scalar ϕ^3 theory. One slight modification needed is that we should use

$$\frac{\bar{m}}{p^2 + m\bar{m}} \tag{3.8}$$

as a propagator. For fermionic coordinates, using the standard Wick-contraction procedure, we get two kinds of interaction vertices in the momentum superspace : Twisted and untwisted vertices. Via Fourier transformation, the Feynman rule is simply to attach a phase factor

$$\exp(-\frac{C^{\alpha\beta}}{2}\pi_{1\alpha}\pi_{2\beta}) \equiv \exp(-\frac{1}{2}\pi_1 \wedge \pi_2)\,, \tag{3.9}$$

for an untwisted vertex and

$$\exp(+\frac{1}{2}\pi_1 \wedge \pi_2)\,, \tag{3.10}$$

for a twisted vertex. But the classification of twisted and untwisted vertex is just a relative matter as in the case of bosonic noncommutativity. As we will see from below, interesting new physics arises from the sectors containing nonplanar diagrams. This is because the planar diagrams do not depend on the deformation parameter except for the star products between the external legs [23].

3.1 Vacuum energy

One loop vacuum diagram vanishes since the diagram is planar.

Similarly as [23], the two loop diagram is evaluated to be

$$\begin{aligned}(\text{bosonic part})\otimes\int d^2\pi_1 d^2\pi_2 d^2\pi_3 \exp\left(-(\pi_1\wedge\pi_2 + \pi_2\wedge\pi_3 + \pi_1\wedge\pi_3)\right)\delta^2(\pi_1+\pi_2+\pi_3) = \\ = (\text{bosonic part}) \otimes \frac{1}{4}\det C\,\delta(0),\end{aligned} \tag{3.11}$$

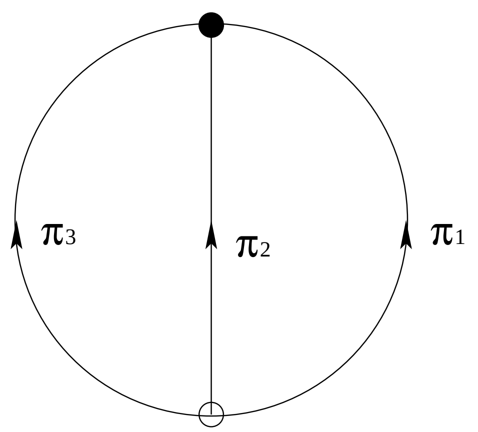

Figure 1: Two loop vacuum diagram. The filled circle denotes a twisted vertex and the unfilled circle an untwisted one.

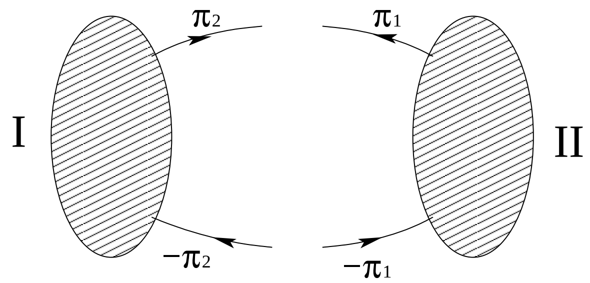

Figure 2: We have cut general vacuum diagrams into two parts.

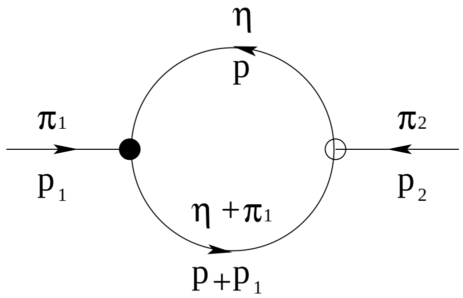

Figure 3: One loop nonplanar diagram with two external lines. Fermionic momenta are denoted in Greek and bosonic momenta in Latin.

where bosonic part is to be

$$(\text{bosonic part}) \sim g^2 \bar{m}^3 \Lambda^2 \,. \tag{3.12}$$

Λ is cutoff. But since $\delta(0) = 0$, (3.11) vanishes.

And this feature continues even when we compute arbitrary higher order loops. The general structure of vacuum diagram is like figure 2. We have cut intermediate lines. Since I or II can be a single line, the cutting is most general. When we join again I and II together to make a single vacuum diagrams, we need to attach $\delta(\pi_1 + \pi_2)$ and $\delta(-\pi_1 - \pi_2)$ at each junction. This gives square of delta function in the form of $\delta^2(\pi_0 + \cdots)$, where π_0 is a loop momenta to be integrated. After integrating out π_0 this factor gives $\delta(0)$ term which is nothing but zero. Stated differently, this is just because there is no single source nor sink of the momentum flow in the closed vacuum diagram. Note that this argument is not true if we attach additional external lines to the vacuum diagram as tadpole diagrams. Anyway, we can conclude that the vacuum energy vanishes up to all higher orders just as $C_{\alpha\beta} = 0$ case. What is surprising here is that there is no algebraical reason for the vacuum energy to vanish. Even though $Q|0\rangle = 0$, it does not guarantee the vanishing of the vacuum energy since $\bar{Q}$ is a broken generator.

3.2 One loop diagrams

Planar diagrams vanish by symmetry just as $C_{\alpha\beta} = 0$ case. Thus we only need to concentrate on nonplanar diagrams. First, we consider explicitly two point and three point vertex functions which are directly related to the effective action.

JHEP12(2003)053

Fermionic parts are integrated into

$$\begin{aligned}\Gamma_{2,F} &= \int d^2\eta \exp\left(-\frac{1}{2}\eta\wedge\pi_1\right)\exp\left(\frac{1}{2}(\eta+\pi_1)\wedge\pi_2\right)\delta(\pi_1+\pi_2)\\ &= \frac{1}{4}(\mathrm{det}C)(\pi_1)^2\delta(\pi_1+\pi_2)\,,\end{aligned}\tag{3.13}$$

which does not vanish unless $C_{\alpha\beta}=0$. Note that the fermionic momentum conservation comes from $\delta(\pi_1+\pi_2)\equiv(\pi_1+\pi_2)^2$. Bosonic part integration gives

$$\begin{aligned}\Gamma_{2,B} &= \bar{m}^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m\bar{m}}\frac{1}{(p+p_1)^2+m\bar{m}}\\ &= \frac{\bar{m}^2}{(4\pi)^2}\left(\log\frac{\Lambda^2}{m\bar{m}}+\cdots\right).\end{aligned}\tag{3.14}$$

One can see that bosonic loop integration is logarithmically divergent. The absence of higher order divergence is the consequence of still-remaining $\mathcal{N}=1/2$ supersymmetry. Combining both bosonic and fermionic parts together, we get

$$S_{\text{eff}} = \int d^4xd^2\pi_1d^2\pi_2\frac{1}{4}\left(\frac{\bar{m}^2}{4\pi^2}\log\frac{\Lambda^2}{m\bar{m}}+\cdots\right)\mathrm{det}C\Phi(x,\pi_1)(\pi_1)^2\Phi(x,\pi_2)\delta(\pi_1+\pi_2)\,.\tag{3.15}$$

We note that $(\pi_1)^2$ corresponds to Q^2 in θ space. Schematically this term can be written as

$$S\sim\int d^4xd^2\theta\Phi Q^2\Phi\,.\tag{3.16}$$

This is still chiral since both Φ and $Q^2\Phi$ are chiral superfields. We see this is a quantum mechanically induced term which is not present in the original classical action. To make the theory renormalizable, we need to add the counter term to cancel this logarithmic divergence. Stated differently, the tree level deformed Wess-Zumino action is not enough quantum mechanically, but we should extend it to accommodate $\Phi Q^2\Phi$ term which cannot be deduced from the $*$ deformations of $\mathcal{N}=1$ supersymmetric action.

Also note that, since the induced term $\int d^2\theta\Phi Q^2\Phi$ gives mass splitting, it is evident that the mass of the fermion and boson is no longer the same. This is another novel feature of the deformed Wess-Zumino model. Phenomenological application of this feature will be an interesting subject of future study.

Three point function is to be evaluated similarly. The fermionic part integration is given by

$$\Gamma_F = \left(\exp\left(-\frac{1}{2}\pi_1\wedge\pi_2\right)+\exp\left(+\frac{1}{2}\pi_1\wedge\pi_2\right)\right)\frac{1}{4}\pi_3\pi_3\mathrm{det}C\delta(\pi_1+\pi_2+\pi_3).\tag{3.17}$$

We see that the phase factor can be expressed as $\Phi*\Phi$ and the the result of loop integration is $\sim\pi\pi$ as that of two point function. Bosonic part integration is

$$\left(\frac{g}{3}\right)^3\int\frac{d^4}{(2\pi)^4}\frac{\bar{m}}{p^2+m\bar{m}}\frac{\bar{m}}{(p+p_1)^2+m\bar{m}}\frac{\bar{m}}{(p+p_1+p_2)^2+m\bar{m}}\,,\tag{3.18}$$

JHEP12(2003)053

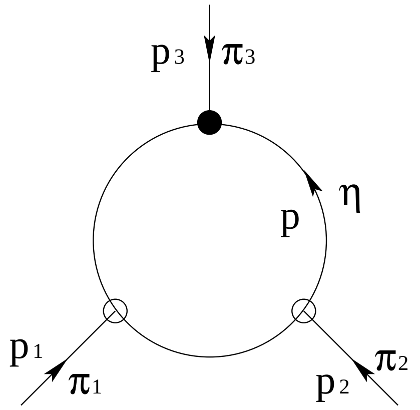

Figure 4: One loop nonplanar diagrams with three external lines.

which is finite. Thus we don't need any further counter terms to cancel infinities. We can see easily that four and higher order point functions are also finite since the loop integration is just the same as scalar ϕ^3 theory.

General structure of fermionic integration of N point function is summarized as

$$\Gamma_{N,F} = \frac{1}{2}\int d^2\pi_0 \sum_{i,j=1}^{n} \exp\left(-\frac{1}{2}\pi_i\wedge\pi_j\right)\delta\left(\sum_{i=1}^{n}\pi_i - \pi_0\right)\otimes\left(\frac{1}{4}(\pi_0)^2\mathrm{det}C\right)\otimes$$
$$\otimes\sum_{i,j=n+1}^{N-n}\exp\left(-\frac{1}{2}\pi_i\wedge\pi_j\right)\delta\left(\sum_{i=1}^{n}\pi_i+\pi_0\right). \qquad (3.19)$$

We see that novel factorization property here. This is the unique property of star product, which is also true for bosonic noncommutativity [24].

If we take $\bar{m}\to\infty$ limit of the action (3.1), bosonic part of the loop integral is simplified quite a lot.

$$\left(\frac{g}{3}\right)^N\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{m}\right)^N\sim\left(\frac{g}{m}\right)^N\Lambda^4. \qquad (3.20)$$

But, this term is badly divergent. Moreover since all N point functions have this divergence, this theory is nonrenormalizable. But in this case, the effective action is factorized into a very simple form, because loop integrations are factored out. The effective action is resummed to be

$$S_{\rm eff} = \frac{1}{2}\int d^4xd^2\pi_0\exp\left(\frac{g}{3m}\Phi(x,\pi_0)\right)_*\left(\frac{1}{4}\pi_0\pi_0\mathrm{det}C\Lambda^4\right)\exp\left(\frac{g}{3m}\Phi(x,-\pi_0)\right)_*, \qquad (3.21)$$

where

$$\exp\left(\Phi(x,\pi)\right)_*\equiv\sum_{l=1}^{\infty}\frac{1}{l!}\int d^2\theta e^{\theta\pi}\left(\Phi(x,\theta)\right)_*^l. \qquad (3.22)$$

This is reminiscent to the expression of the 1-loop effective action of bosonic noncommutative field theories, where effective actions can be resummed as interactions between two open Wilson lines[24]. Even though this structure looks interesting with relation to the open-closed string duality, further study is needed to understand the full meaning in terms of string theory.

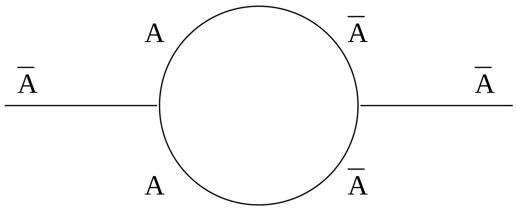

Figure 5: $\bar{A}\bar{A}$ term which is induced quantum mechanically.

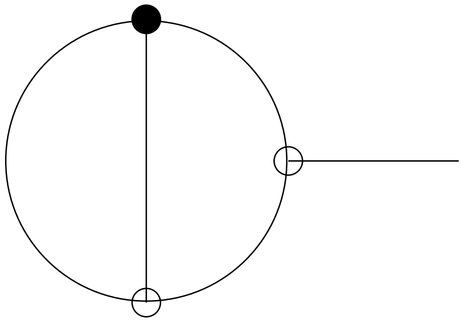

Figure 6: An example of nonplanar tadpole diagram.

3.3 Component fields

If $\bar{g} = 0$ limit is taken, the potential (3.4) becomes

$$V = m\bar{m}A\bar{A} + g\bar{m}A^2\bar{A} + \frac{1}{3}g\bar{m}^3\det C\bar{A}^3\,. \tag{3.23}$$

We can see very easily that this potential induces a quantum mechanical term

$$S_{\text{eff}} \sim \int d^4x\bar{A}(x)\bar{A}(x)\,, \tag{3.24}$$

which can be expressed in terms of superfield

$$S_{\text{eff}} \sim \int d^4xd^2\theta\Phi(x,\theta)Q^2\Phi(x,\theta)\,. \tag{3.25}$$

Thus again we see that $\int \Phi Q^2\Phi$ is needed for quantum completeness.

3.4 Tadpole contributions

We have seen that vacuum diagrams vanish in general even though it is nonplanar. This is caused by momentum conservation $\delta^2(\cdots)$ factor attached. But for tadpole diagrams, there is no more $\delta^2(\cdots)$ factor since tadpoles have outer source for momentum flow. Thus nonplanar tadpole diagrams do not vanish in general. We will present explicit examples here.

Bosonic loop integration gives $g^3\bar{m}^4\log\frac{\Lambda^2}{m\bar{m}}$. With fermionic integration included, the effective action is

$$S_{\text{eff}} \sim \int d^4xd^2\pi_0\left(g^3\bar{m}^4\log\frac{\Lambda^2}{m\bar{m}}\right)\det C\delta(\pi_0)\Phi(x,\pi_0)\,. \tag{3.26}$$

Generally speaking, we need redefinition of fields to cancel liner terms. Usually such kinds of field redefinition give nonzero cosmological constant. But will it be the case also here?

This tadpole term amounts to adding SF in componentwise to the classical potential (3.2). Here $S \sim g^3\bar{m}^4 \log\frac{\Lambda^2}{m\bar{m}}\det C$. Then the general form of the modified potential becomes

$$V_{\text{modified}} = -F\bar{F} - F(mA + gA^2 + S) - H(F) - \frac{g}{3}\det CF^3 - \bar{m}\bar{A}\bar{F}\,. \tag{3.27}$$

Here we added a term $H(F)$ coming from one loop computation since we are calculating two loop diagrams. Note that $H(F)$ is a function of F only and there is no $\bar{F}$ correction. It is easy to see that, to eliminate the linear term, we need to redefine $A \to A + T$ where T satisfies the equation $gT^2 + mT + S = 0$. This equations always has a solution. Thus the tadpole term does not induce any cosmological constant. We should note that this is true because only A needs to be redefined but $\bar{A}$ remain intact. This is possible because we are treating Euclidean theories.

4. Discussions

Considering quantum fluctuations of field theories in noncommutative superspace, we get additional terms as $\int d^2\theta\Phi Q^2\Phi$ which is not present in the original $*$-deformed Wess-Zumino model. Thus we can say that the star deformation of the noncommutative superspace is not quantum mechanically complete, but we need to extend the model to accommodate such operators. We have shown the possibility of this extension since the set of $\mathcal{N} = 1/2$ supersymmetric action in general is larger than the set of $*$-deformation of $\mathcal{N} = 1$ supersymmetric action. From the string theory point of view, it is quite interesting to ask whether such kind of extension is also natural when we consider gauge theories in the noncommutative superspace derived from string theory. Especially it will be interesting to trace the the possible stringy origin of the terms induced by quantum fluctuation.

Dijkgraaf-Vafa theory [20, 21] can be considered in this setting, since even though there is no "holomorphicity", there is the notion of "anti-holomorphicity". Actually, considering the superpotential for the case of $\bar{g} = 0$, we see that there is no $\bar{m}$ dependence for the planar diagrams. The bosonic and fermionic integrations cancel each other. But as we have shown explicitly in the section 3, the contributions from nonplanar graphs are not zero. Since there is no cancellation between bosonic and fermionic integrations, the effective superpotential depends on $\bar{m}$. Thus to fully understand the quantum structure including nonplanar diagrams, we need further study in this direction.

Acknowledgments

We thank Jan de Boer for discussions. This work is supported in part by the Stichting FOM.

Note added: As this article was being completed, we received the preprint [25] which partly overlaps with the present work.

JHEP12(2003)053

References

[1] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[2] S. Ferrara and M.A. Lledo, *Some aspects of deformations of supersymmetric field theories*, *J. High Energy Phys.* **05** (2000) 008 [hep-th/0002084].

[3] S. Terashima, *A note on superfields and noncommutative geometry*, *Phys. Lett.* **B 482** (2000) 276 [hep-th/0002119].

[4] R. Casalbuoni, *Relativity and supersymmetries*, *Phys. Lett.* **B 62** (1976) 49.

[5] R. Casalbuoni, *On the quantization of systems with anticommutating variables*, *Nuovo Cim.* **A33** (1976) 115.

[6] R. Casalbuoni, *The classical mechanics for Bose-Fermi systems*, *Nuovo Cim.* **A33** (1976) 389.

[7] J.H. Schwarz and P. Van Nieuwenhuizen, *Speculations concerning a fermionic substructure of space-time*, *Nuovo Cim.* **34** (1982) 21.

[8] D. Klemm, S. Penati and L. Tamassia, *Non(anti)commutative superspace*, *Class. and Quant. Grav.* **20** (2003) 2905 [hep-th/0104190].

[9] R. Abbaspur, *Generalized noncommutative supersymmetry from a new gauge symmetry*, hep-th/0206170.

[10] J. de Boer, P.A. Grassi and P. van Nieuwenhuizen, *Non-commutative superspace from string theory*, *Phys. Lett.* **B 574** (2003) 98 [hep-th/0302078].

[11] H. Ooguri and C. Vafa, *Gravity induced C-deformation*, hep-th/0303063.

[12] H. Ooguri and C. Vafa, *The C-deformation of gluino and non-planar diagrams*, *Adv. Theor. Math. Phys.* **7** (2003) 53 [hep-th/0302109].

[13] H. Kawai, T. Kuroki and T. Morita, *Dijkgraaf-Vafa theory as large-N reduction*, *Nucl. Phys.* **B 664** (2003) 185 [hep-th/0303210].

[14] I. Chepelev and C. Ciocarlie, *A path integral approach to noncommutative superspace*, *J. High Energy Phys.* **06** (2003) 031 [hep-th/0304118].

[15] J.R. David, E. Gava and K.S. Narain, *Konishi anomaly approach to gravitational F-terms*, *J. High Energy Phys.* **09** (2003) 043 [hep-th/0304227].

[16] N. Seiberg, *Noncommutative superspace, $N = 1/2$ supersymmetry, field theory and string theory*, *J. High Energy Phys.* **06** (2003) 010 [hep-th/0305248].

[17] N. Berkovits and N. Seiberg, *Superstrings in graviphoton background and $N = 1/2 + 3/2$ supersymmetry*, *J. High Energy Phys.* **07** (2003) 010 [hep-th/0306226].

[18] T. Asakawa, S. Sugimoto and S. Terashima, *Exact description of D-branes via tachyon condensation*, *J. High Energy Phys.* **02** (2003) 011 [hep-th/0212188].

[19] K.A. Intriligator and N. Seiberg, *Lectures on supersymmetric gauge theories and electric-magnetic duality*, *Nucl. Phys.* **45BC** *(Proc. Suppl.)* (1996) 1 [hep-th/9509066].

[20] R. Dijkgraaf and C. Vafa, *Matrix models, topological strings and supersymmetric gauge theories*, *Nucl. Phys.* **B 644** (2002) 3 [hep-th/0206255].

[21] R. Dijkgraaf and C. Vafa, *A perturbative window into non-perturbative physics*, hep-th/0208048.

[22] R. Dijkgraaf, M.T. Grisaru, C.S. Lam, C. Vafa and D. Zanon, *Perturbative computation of glueball superpotentials*, *Phys. Lett.* **B 573** (2003) 138 [hep-th/0211017].

[23] S. Minwalla, M. Van Raamsdonk and N. Seiberg, *Noncommutative perturbative dynamics*, *J. High Energy Phys.* **02** (2000) 020 [hep-th/9912072].

[24] Y. Kiem, S.J. Rey, H.T. Sato and J.T. Yee, *Open Wilson lines and generalized star product in nocommutative scalar field theories*, *Phys. Rev.* **D 65** (2002) 026002 [hep-th/0106121].

[25] R. Britto, B. Feng and S.J. Rey, *Deformed superspace, $N = 1/2$ supersymmetry and (non)renormalization theorems*, *J. High Energy Phys.* **07** (2003) 067 [hep-th/0306215].